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By
H. Kouts
K. Downes
G. Price
R. Sher

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Brookhaven National Laboratory
Upton, New York

Technical Information Service, Oak Ridge, Tennessee

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A MEASUREMENT OF NEUTRON TEMPERATURE IN A URANIUM ROD-WATER MODERATED LATTICE

By H. Kouts, K. Downes, G. Price, and R. Sher

Abstract: The relative danger coefficients of boron and cadmium have been measured in an assembly of 1.15% enriched uranium rods in ordinary water, at a water to uranium volume ratio of 3. The experiment consisted in finding the relative effects on neutron multiplication of measured amounts of boron and cadmium in the water moderator. The ratio of the observed danger coefficients is a measure of the ratio of the cross-sections of the two poisons, and since the two have very different cross-section curves in the thermal range, a basis for the estimation of a neutron temperature exists. The measurement resembles somewhat one done by G. B. Gavin (KAPL - 1142).

Under the assumption that the thermal neutrons have a Maxwell distribution of velocities, the measurement implies a characteristic temperature of .0262 ± .0014 volts (304° ± 16° °K). The water temperature at the time of the measurement was 297 °K.

Experimental Methods: A lattice of 1.15% enriched uranium rods, .600" diameter, was loaded in a light water moderator until a multiplication of about 500 was reached. The method of loading, and the safety precautions were as described in BNL Log No. C-7605 (Safety of Subcritical Loadings in T-526), except that now the console and instrumentation have been greatly improved. For instance, during the measurement described in this report the flux levels were monitored by six separate detectors and channels of instrumentation, and the safety rod

was set to trip on any one of four flux level monitors. In addition, a manual rod trip and a manual water dump were available.

After this predetermined stopping point had been reached, the source was removed, and the count rate from multiplication of spontaneous fission neutrons was measured. A measured amount of boric acid solution was added to the water moderator, after the equivalent volume of pure water was removed, and the count rate was again measured. A measured amount of cadmium sulphate solution was then added (again after removal of the same volume of moderator), and the count rate was measured a third time.

Three counters were used to measure the flux levels at each stage. One was a small BF_3 counter at the center of a triangular lattice cell; the second was a small enriched uranium fission chamber at the center of a triangular lattice cell, and the third was a fission chamber located inside a fuel rod. Three detectors were used in order that independent checks on the results might be had; they also increased the statistical accuracy, in that the determination of relative flux level would be based on adding up the count rates from all counters.

While the flux level measurements were made, the water temperature was monitored by means of a chromel-alumel thermocouple.

Analysis: The reciprocal of the observed count rate is, for loadings sufficiently near critical, linear in k_{eff}:

$$\frac{1}{R} \propto 1 - k_{eff} = 1 - k_{\infty} e^{-M^2 B^2_g}$$

If a small amount of poison is added to the moderator, as was done here, k_{eff} will

change because of the effect on f. For sufficiently small increments, the change in k_{eff} is proportional to the macroscopic cross-section of the poison in the moderator:

$$\delta\, k_{eff} \propto \Sigma_{poison}$$

and we suppose this relation to hold for the poison concentrations used in this measurement. We let the subscripts 0, 1, 2 refer respectively to situations where the moderator was pure water, where it contained boron, and where it contained boron and cadmium. Further, we let the changes in k_{eff} produced by the addition of the boron and the cadmium be respectively $-\delta$ and $-\Delta$. Then

$$\frac{1}{R_o} \propto 1 - k_o = 1 - k_1 - \delta$$

$$\frac{1}{R_1} \propto 1 - k_1$$

$$\frac{1}{R_2} \propto 1 - k_2 = 1 - k_1 + \Delta$$

Then

$$\frac{1}{R_1} - \frac{1}{R_o} \propto \delta \propto \Sigma_B$$

$$\frac{1}{R_2} - \frac{1}{R_1} \propto \Delta \propto \Sigma_{cd}$$

with the same constants of proportionality in both relations.

Therefore

$$\frac{\frac{1}{R_2} - \frac{1}{R_1}}{\frac{1}{R_1} - \frac{1}{R_o}} = \frac{\Sigma_{cd}}{\Sigma_B}$$

Again, the macroscopic cross-sections are proportional to the mole fractions M times the microscopic cross-sections, with Avogodro's number the constant of

proportionality. Thus

$$\frac{\frac{1}{R_2} - \frac{1}{R_1}}{\frac{1}{R_1} - \frac{1}{R_o}} = \frac{M_{cd}\ \sigma_{cd}}{M_B\ \sigma_B}$$

The cross-sections to be used in this expression are averages over the energy distribution of the thermal neutrons in the lattice. If we assume the flux to have a Maxwell energy distribution

$$\phi(E)\, dE = \frac{1}{(kT)^2} E\, e^{-E/kT}\, dE$$

then the averages are

$$\sigma_B = \int_0^\infty dE\, \phi(E) \quad \sigma_B(E) = c_1 \int_0^\infty dE\ E^{1/2} e^{-E/kT}$$

$$\sigma_{cd} = \int_0^\infty dE\, \phi(E) \quad \sigma_{cd}(E) = c_2 \int_0^\infty dE\, \frac{E^{1/2}\, e^{-E/kT}}{(E - E_o)^2 + \Gamma^2/4}$$

The boron cross-section we take as 1/v, with $c_1 = 118.6$. This corresponds to the latest cross-section value of 750 barns at .025 e.v. The cadmium cross-section is by recent data* characterized by $E_o = 0.176$ e.v., $\Gamma = 0.115$ e.v., and a cross-section of 7700 barns at the peak of the resonance. Accordingly, c_2 is assumed to be 10.680.

The boron cross-section can be integrated immediately, to give

$$\sigma_B = \frac{105.1}{\sqrt{kT}} \text{ barns}$$

Integration of the expression for the cadmium cross-section is more involved. It was accomplished in two ways - by numerical integration for several values of kT, and by an analytic approximation. This latter method involves transforming the integral to

* J. Harvey, Private communication.

$$\sigma_{cd} = c_2 \int_{-\infty}^{\infty} \frac{u^2 \, e^{-u^2/kT}}{(u^2 - E_o)^2 + \Gamma^2/4} \, du$$

then (using Paarseval's theorem on the convolution of two Fourier transforms) into the form

$$\sigma_{cd} = c_2 \int_{-\infty}^{\infty} dt \; F\left(e^{-\frac{u^2}{kT}}\right) \; F\left(\frac{u^2}{(u^2 - E_o)^2 + \Gamma^2/4}\right)$$

The symbol "F" means here "Fourier transform of". This latter form can again be transformed to lead to error functions of complex argument. These in turn lead to expressions of the form

$$\int_0^a e^{v^2} \sin b\, v \, d\, v, \quad \int_0^a e^{v^2} \cos b\, v \, d\, v$$

These integrals can be evaluated by means of an approximation valid over the entire range of the argument to better than one part in 10^5:

$$e^{v^2} \simeq \frac{1}{\sqrt{\pi}} \left\{ 1 + 2 \sum_{n=1}^{\infty} e^{-n^2} \cosh 2 n v \right\} *$$

Finally, all integrals involve only tabulated functions (trigonometric, hyperbolic, and error functions of real argument).

The calculation is only sketched, because of its length. The two independent evaluations of σ_{cd} (numerical and analytic) agree to better than 1%.

The curves of σ_B, σ_{cd}, and $\frac{\sigma_{cd}}{\sigma_B}$ are given as figures 1, 2, and 3.

Prior to the poisoning of the moderator, a curve of the spontaneous fission multiplication was run as a function of rod loading. From this the critical

* See H. E. Salzer, Mathematical Tables and other aids to computation, pp. 67-70 (1951).

mass was determined. This permitted rough estimation of the change in k_{eff} caused by the poison. We found that adding the boron changed k_{eff} by about 3.6×10^{-3}; the effect of adding cadmium was to reduce k_{eff} again by about 2.8×10^{-3}. These changes are certainly small enough to permit assuming linearity between δk_{eff} and Σ_{poison}.

Results: The observed count rates are given in table I. By the methods of the preceding section, we calculate for the three cases:

BF_3 counter: $\frac{\Delta}{\delta} = .780 \pm .035$

Fission chamber in moderator: $\frac{\Delta}{\delta} = .840 \pm .053$

Fission chamber in fuel rod: $\frac{\Delta}{\delta} = .754 \pm .049$

Counters summed: $\frac{\Delta}{\delta} = .790 \pm .022$

The mole fractions used were respectively .09332 of cadmium and .5587 of boron. Thus

$$\frac{.09332 \ \sigma_{cd}}{.5587 \ \sigma_B} = .790 \pm .022$$

$$\frac{\sigma_{cd}}{\sigma_B} = 4.73 \pm .17$$

From figure 3 we get, then,

$$kT = .0262 \pm .0014 \text{ ev}$$

$$= 304^\circ \pm 16^\circ \text{ K}$$

The water temperature during the measurement was 297° K. Thus the neutron temperature was $7^\circ \pm 16^\circ$ above the moderator temperature.

The probable error cited is based solely on statistical considerations. There is a possible additional source of error, from incorrect cross-sections. We estimate that this uncertainty could increase the error by at most .0006 electron volts, or 7° K.

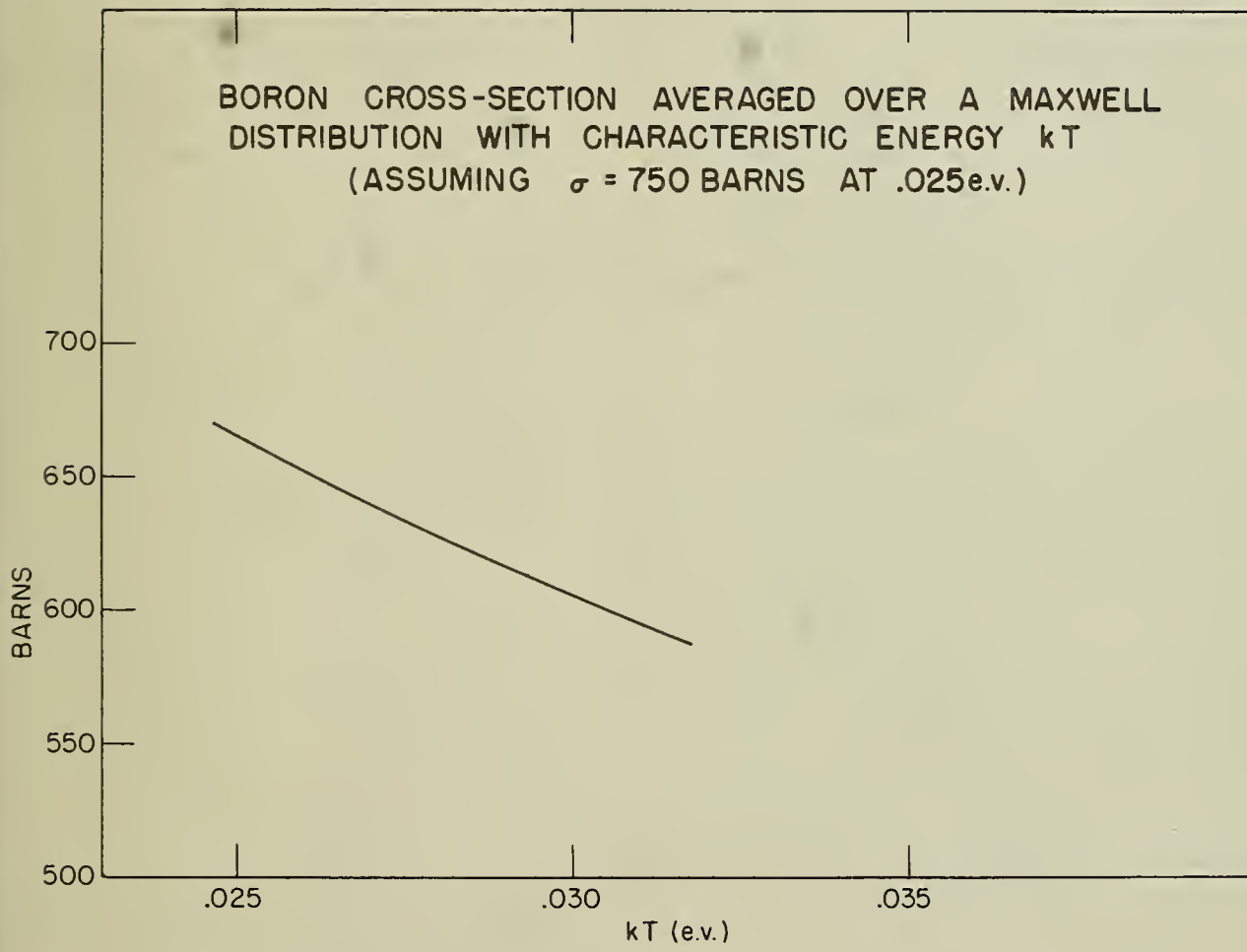
BORON CROSS-SECTION AVERAGED OVER A MAXWELL DISTRIBUTION WITH CHARACTERISTIC ENERGY kT
(ASSUMING σ = 750 BARNS AT .025e.v.)
700
650
600
550
500
BARNS
.025
.030
.035
kT (e.v.)

Fig. 1

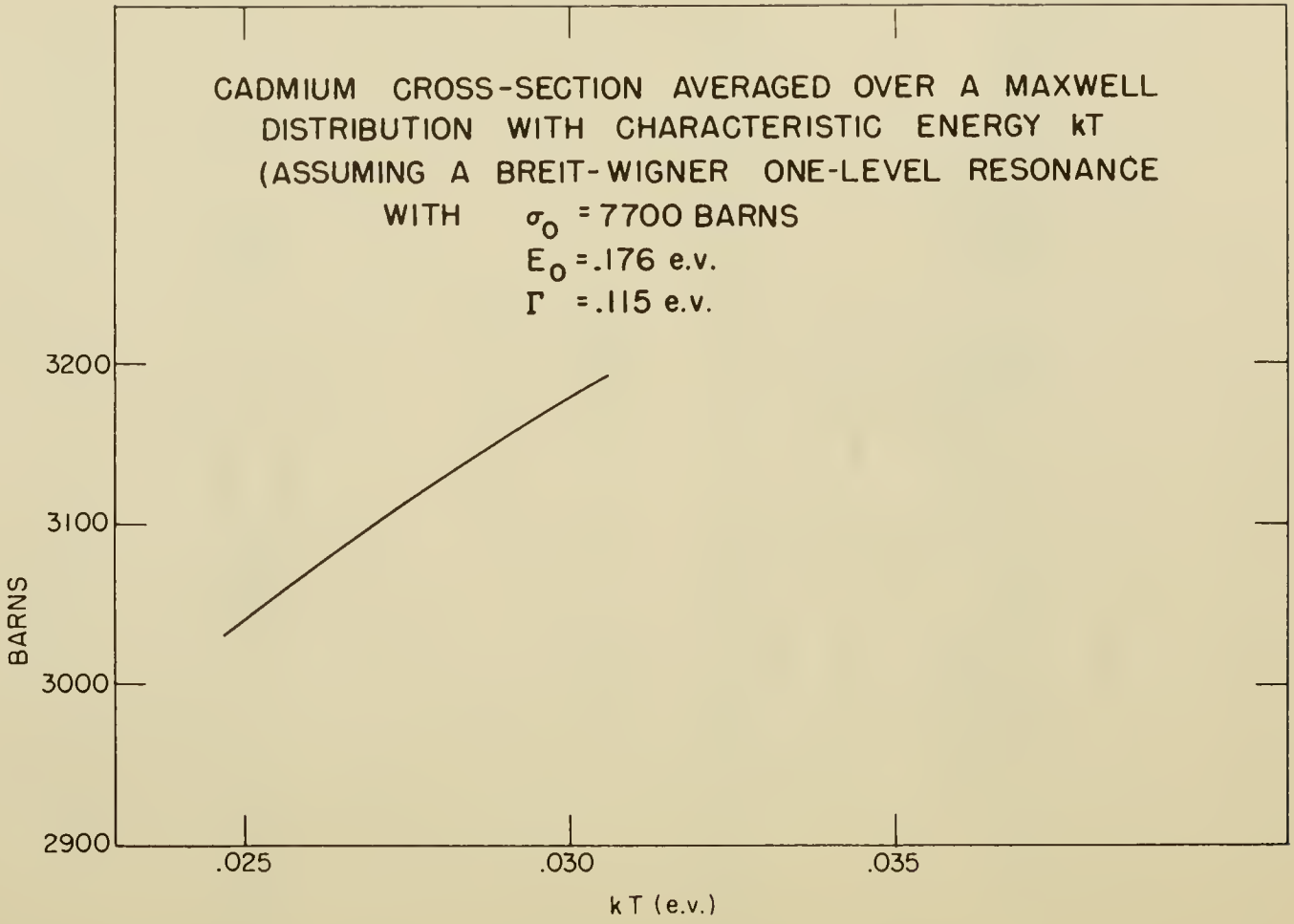
CADMIUM CROSS-SECTION AVERAGED OVER A MAXWELL DISTRIBUTION WITH CHARACTERISTIC ENERGY kT
(ASSUMING A BREIT-WIGNER ONE-LEVEL RESONANCE
WITH σ_0 = 7700 BARNS
E_0 = .176 e.v.
Γ = .115 e.v.
3200
3100
3000
2900
BARNS
.025
.030
.035
kT (e.v.)

Fig. 2

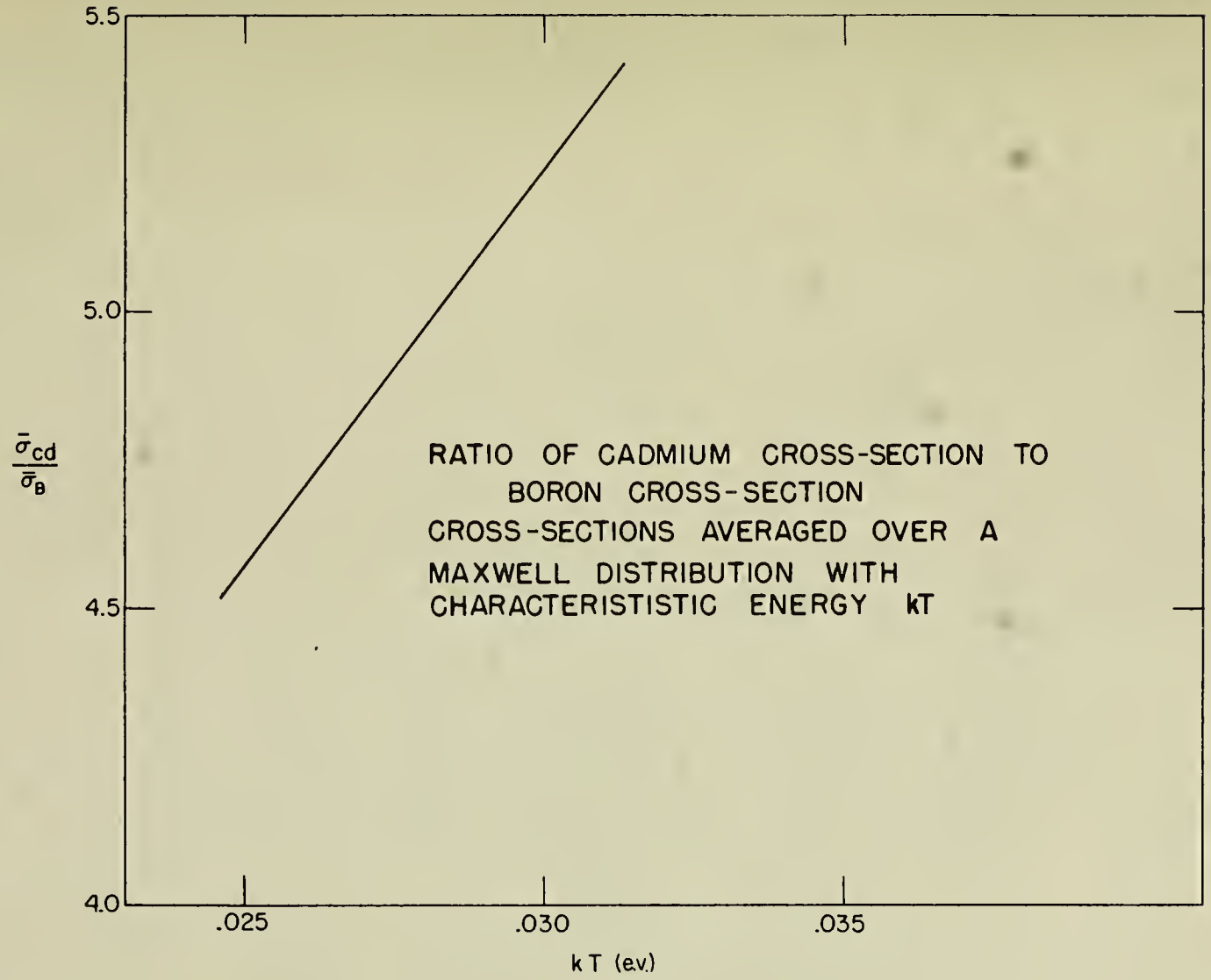
5.5
5.0
4.5
4.0
$\frac{\bar{\sigma}_{cd}}{\bar{\sigma}_B}$
RATIO OF CADMIUM CROSS-SECTION TO
BORON CROSS-SECTION
CROSS-SECTIONS AVERAGED OVER A
MAXWELL DISTRIBUTION WITH
CHARACTERISTISTIC ENERGY kT
.025
.030
.035
kT (ev.)

Fig. 3

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